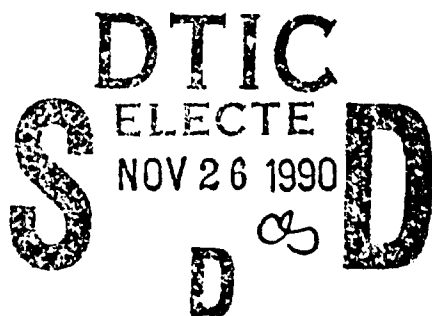RADC-TR-90-251
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NUMERIC AND SYMBOLIC SIGNAL PROCESSING FOR VERY HIGH SUBCLUTTER VISIBILITY

Syracuse University

Dr. Donald Weiner and Braham Himed

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Rome Air Development Center
Air Force Systems Command
Griffiss Air Force Base, NY 13441-5700

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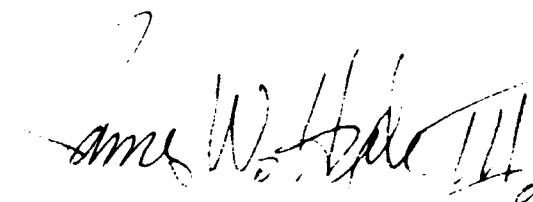
APPROVED:

VINCENT VANNICOLA
Project Engineer

APPROVED:

JAMES W. YOUNGBERG, LT COL, USAF
Deputy Director of Surveillance

FOR THE COMMANDER:

JAMES W. HYDE, III
Directorate of Plans & Programs

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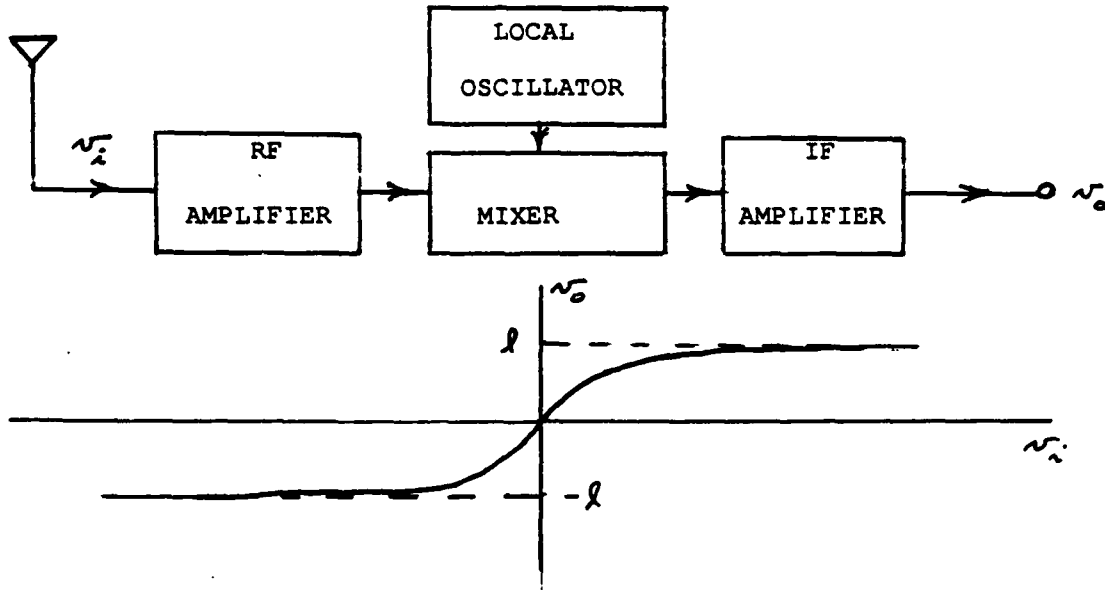
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1. SYSTEM MODEL

The front end of the radar receiver is shown below:

Let the voltage transfer function relating the IF amplifier output signal to the RF amplifier input signal be the zero-memory nonlinearity specified by

$$v_o = g\,(v_i) = \frac{2\ell}{\sqrt{2\pi\sigma_g^2}} \int_0^{v_i} e^{-Z^2/2\sigma_g^2}\, dZ. \qquad (1)$$

Due to the mixer, the carrier frequency of v_o is translated from that of v_i by an amount equal to the local oscillator frequency. Other than for this frequency shift, the spectrum and correlation function of v_o is unaltered. For simplicity, the nonlinearity is modeled as though v_o and v_i have identical carrier frequencies. The voltage transfer function accounts for the cumulative nonlinearities in the RF amplifier, mixer, and IF amplifier.

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Observe that

$$g(\infty) = \frac{2\ell}{\sqrt{2\pi\sigma_g^2}} \int_0^\infty e^{-z^2/2\sigma_g^2}\, dZ = \frac{2\ell}{\sqrt{2\pi\sigma_g^2}}\, \sigma_g \int_0^\infty e^{-y^2/2}\, dy = \ell \qquad (2)$$

and

$$g(-\infty) = \frac{2\ell}{\sqrt{2\pi\sigma_g^2}} \int_0^{-\infty} e^{-z^2/2\sigma_g^2}\, dZ = -\frac{2\ell}{\sqrt{2\pi\sigma_g^2}} \int_{-\infty}^0 e^{-z^2/2\sigma_g^2}\, dZ = -\ell. \qquad (3)$$

Hence, the voltage transfer function is modeled as a soft nonlinearity with saturation levels at $\pm\ell$. Note that the integrand of $g(\cdot)$ is proportional to a zero-mean Gaussian probability density function (pdf) with variance σ_g^2. σ_g is referred to as the _rms_ _value_ _of_ _the_ _voltage_ _transfer_ _function_. Let σ_i denote the rms value of v_i. The ratio of these two rms values yields the parameter

$$\alpha = \frac{\sigma_g}{\sigma_i}. \qquad (4)$$

When $v_i = \sigma_g$, the output is

$$v_o = g(\sigma_g) = \frac{2\ell}{\sqrt{2\pi\sigma_g^2}} \int_0^{\sigma_g} e^{-z^2/2\sigma_g^2}\, dZ = \frac{2\ell}{\sqrt{2\pi}} \int_0^1 e^{-y^2/2}\, dy \approx 0.68\ell. \qquad (5)$$

The slope of the voltage transfer function is

$$\frac{dv_o}{dv_i} = \frac{2\ell}{\sqrt{2\pi\sigma_g^2}}\, e^{-v_i^2/2\sigma_g^2}. \qquad (6)$$

At the origin, therefore, the slope is $\sqrt{\frac{2}{\pi}}\, \frac{\ell}{\sigma_g}$. Consider the linear transfer function,

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$$v_o = f(v_i) = \sqrt{\frac{2}{\pi}} \frac{\ell}{\sigma_g} v_i. \tag{7}$$

When $v_i = \sigma_g$, the output of the linear transfer function is

$$\tag{8}$$

$$v_o = \sqrt{\frac{2}{\pi}} \ell \approx 0.80 \ell.$$

Hence, when $v_i = \sigma_g$, the nonlinear transfer function is down from an ideal linear transfer function by

$$20 \log_{10} \frac{.68\ell}{.80\ell} \approx -1.4 \quad dB. \tag{9}$$

In this sense, σ_g is a measure of the linearity of the voltage transfer function. On the other hand, σ_i is a measure of the input fluctuations. Thus, α is an indicator of the "width" of the linear portion of the voltage transfer function relative to the rms value of the input signal.

2. DISTORTION OF THE PDF CAUSED BY THE NONLINEARITY

Let the pdf's of V_i and V_o be denoted by $p_{V_i}(v_i)$ and $p_{V_o}(v_o)$, respectively. It follows that

$$p_{V_o}(v_o) = p_{V_i}(v_i) \left| \frac{dv_i}{dv_o} \right| \Bigg|_{v_i = g^{-1}(v_o)}. \tag{10}$$

From Page 2,

$$\frac{dv_i}{dv_o} = \frac{\sqrt{2\pi\sigma_g^2}}{2\ell} e^{+v_i^2/2\sigma_g^2}. \tag{11}$$

Let V_i be a zero-mean Gaussian random variable with variance σ_i^2.

Then

$$P_{V_i}(v_i) = \frac{1}{\sqrt{2\pi\sigma_i^2}}\, e^{-v_i^2/2\sigma_i^2} \ . \tag{12}$$

It follows that

$$P_{V_o}(v_o) = \frac{1}{\sqrt{2\pi\sigma_i^2}}\,\frac{\sqrt{2\pi\sigma_g^2}}{2\ell}\ e^{(v_i^2/2\sigma_g^2)-(v_i^2/2\sigma_i^2)}\bigg|_{v_i=g^{-1}(v_o)} \ . \tag{13}$$

Simplifying, we have

$$\tag{14}$$
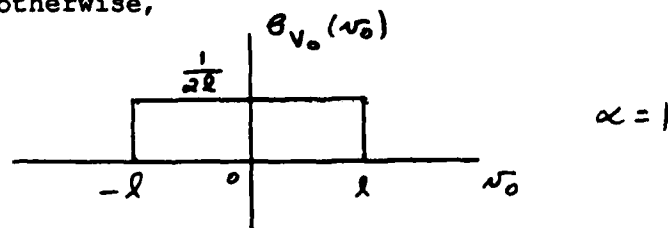$$P_{V_o}(v_o) = \frac{\sigma_g}{\sigma_i}\,\frac{1}{2\ell}\ e^{-(v_i^2/2\sigma_g^2)\cdot[\sigma_g^2/\sigma_i^2 - 1]}\bigg|_{v_i=g^{-1}(v_o)}$$

$$= \frac{\alpha}{2\ell}\ e^{-(v_i^2/2\sigma_g^2)[\alpha^2-1]}\bigg|_{v_i=g^{-1}(v_o)} \tag{15}$$

$$= \frac{\alpha}{2\ell}\ e^{-[(\alpha^2-1)/2\sigma_g^2][g^{-1}(v_o)]^2} \ . \tag{16}$$

For $\alpha = 1$,

$$P_{V_o}(v_o) = \begin{cases} \frac{1}{2\ell} \ ; & -\ell \le v_o \le \ell \\[2mm] 0 \ ; & \text{otherwise,} \end{cases} \tag{17}$$



Thus, the IF amplifier output is uniformly distributed when $\sigma_i = \sigma_g$.
For $\alpha > 1$, the exponent in $P_{V_o}(v_o)$ is negative. Note that

$$g^{-1}(\ell) = \infty \quad \text{and} \quad g^{-1}(-\ell) = -\infty \ . \tag{18}$$

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Thus, the density function goes to zero at $v_o = \pm \ell$. The output density function is <u>approximately</u> Gaussian provided $\alpha \gg 1$. However, irrespective of how large is α, $p_{V_o}(v_o)$ can never be Gaussian since v_o is constrained to the interval $(-\ell, \ell)$ whereas a Gaussian random variable can assume values in the infinite interval $(-\infty, \infty)$. For $\alpha < 1$, the exponent in $p_{V_o}(v_o)$ is positive. The density function then goes to ∞ for $v_o = \pm \ell$. In the limit, as $\alpha \to 0$, the density function approaches two impulses as shown below.
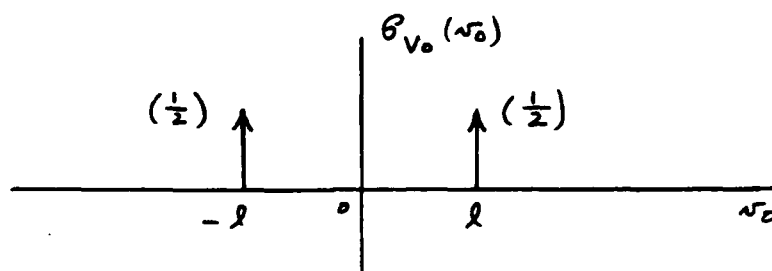
This suggests that the characteristic of the voltage transfer function approaches that of a hard limiter as $\alpha \to 0$:
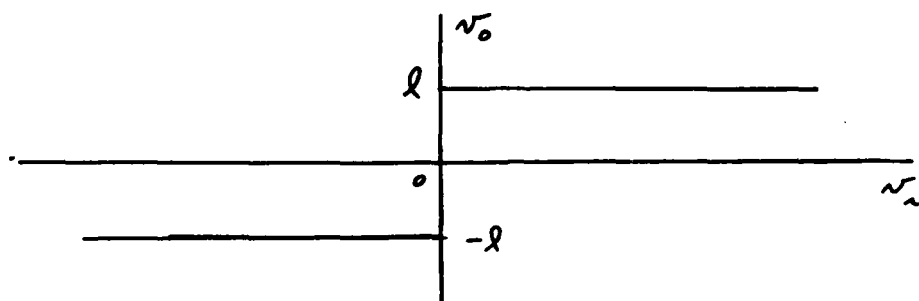
Sketches of $p_{V_o}(v_o)$ for various values of $\alpha = \dfrac{\sigma_q}{\sigma_i}$ are shown below:

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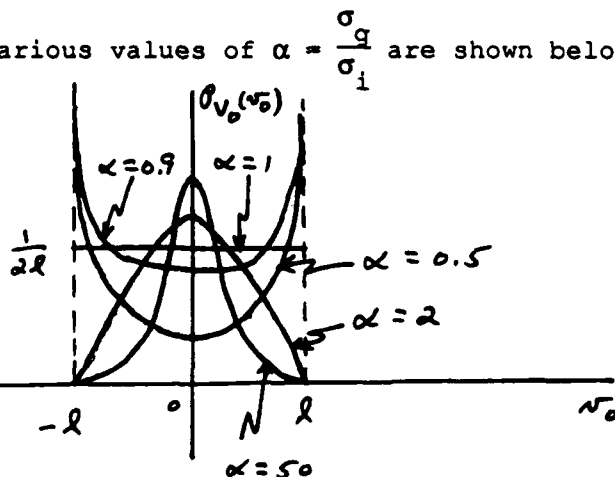
At the origin, $g^{-1}(0) = 0$ and

$$p_{V_o}(v_o) = \frac{\alpha}{2\ell} . \tag{19}$$

3. DISTORTION OF THE CORRELATION FUNCTION CAUSED BY THE NONLINEARITY

Define the Fourier transform of the voltage transfer function to be

$$G(ju) = \int_{-\infty}^{\infty} g(v_i) e^{-juv_i} dv_i . \tag{20}$$

The output correlation function is given by

$$R_o(\tau) = E[v_o(t) v_o(t-\tau)] . \tag{21}$$

The output voltages can be expressed in terms of the Fourier transform of the voltage transfer function according to

$$v_o(t) = g[v_i(t)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(ju) e^{juv_i(t)} du \tag{22}$$

and

$$v_o(t-\tau) = g[v_i(t-\tau)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(jw) e^{jwv_i(t-\tau)} dw. \tag{23}$$

It follows that

$$R_o(\tau) = E\left[\frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(ju) G(jw) e^{juv_i(t)} e^{jwv_i(t-\tau)} du\, dw\right] \tag{24}$$

$$= \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(ju) G(jw) E\left[e^{juv_i(t)} e^{jwv_i(t-\tau)}\right] du\, dw. \tag{25}$$

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The quantity,

$$M(ju, jw) = E\left[e^{juv_i(t)} e^{jwv_i(t-\tau)}\right], \qquad (26)$$

is recognized as the joint characteristic function of the random variables $v_i(t)$ and $v_i(t-\tau)$. Hence,

$$R_o(\tau) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(ju)\ G(jw)\ M(ju, jw)\ du\ dw. \qquad (27)$$

Let $v_i(t)$ and $v_i(t-\tau)$ be jointly Gaussian zero-mean random variables with variance σ_g^2 and correlation function

$$R_i(\tau) = E[v_i(t)\ v_i(t-\tau)]. \qquad (28)$$

Their joint characteristic function is then given by

$$\qquad (29)$$

$$M(ju, jw) = e^{-1/2[\sigma_i^2 u^2 + 2R_i(\tau) u w + \sigma_i^2 w^2]}$$

$$\qquad (30)$$

$$= e^{-(\sigma_i^2/2)\cdot[u^2 + 2\rho_i(\tau) u w + w^2]}$$

where

$$\rho_i(\tau) = \frac{R_i(\tau)}{\sigma_i^2} \qquad (31)$$

is the correlation coefficient relating $v_i(t)$ and $v_i(t-\tau)$. As a result, the output correlation function can be expressed as

$$R_o(\tau) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(ju)\ G(jw)\ e^{-(\sigma_i^2/2)[u^2 + w^2] - R_i(\tau) uw}\ dudw. \qquad (32)$$

To evaluate G(ju), we make use of the differentiation property of the

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Fourier transform. To begin with, we have the Fourier transform pair

$$g(v_i) = \frac{2\ell}{\sqrt{2\pi\sigma_g^2}} \int_0^{v_i} e^{-z^2/2\sigma_g^2} \, dZ \quad \longleftrightarrow \quad G(ju).$$ (33)

Using the differentiation property,

$$\frac{dg(v_i)}{dv_i} = \frac{2\ell}{\sqrt{2\pi\sigma_g^2}} e^{-v_i^2/2\sigma_g^2} \quad \longleftrightarrow \quad ju \, G(ju).$$ (34)

However, it is well known that

$$e^{-v_i^2/2\sigma_g^2} \quad \longleftrightarrow \quad \sqrt{2\pi\sigma_g^2} \, e^{-\sigma_g^2 u^2/2}.$$ (35)

It follows that

$$\frac{2\ell}{\sqrt{2\pi\sigma_g^2}} \sqrt{2\pi\sigma_g^2} \, e^{-\sigma_g^2 u^2/2} = ju \, G(ju).$$ (36)

Thus,

$$G(ju) = \frac{2\ell}{ju} e^{-\sigma_g^2 u^2/2}.$$ (37)

Substituting into the expression for $R_o(\tau)$, we have

$$R_o(\tau) = \frac{(2\ell)^2}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} -(1/uw) e^{-(\sigma_g^2/2)[u^2 + w^2]} e^{-(\sigma_i^2/2)[u^2 + w^2] - R_i(\tau)uw} \, du \, dw.$$ (38)

Combining terms in the exponent of the integrand,

$$R_o(\tau) = -\frac{(2\ell)^2}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (1/uw) e^{-[(\sigma_g^2+\sigma_i^2)/2](u^2 + w^2) - R_i(\tau)uw} \, du \, dw.$$ (39)

To obtain an integrand without the factor of $\frac{1}{uw}$, we differentiate $R_o(\tau)$ with respect to $R_i(\tau)$. Specifically,

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$$\frac{dR_o(\tau)}{dR_i(\tau)} = \frac{(2\ell)^2}{4\pi^2} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} e^{-[(\sigma_g^2+\sigma_g^2)/2].(u^2+w^2) - R_i(\tau)uw} \, dudw.$$

(40)

Now make the change of variables

$$x = \sqrt{\frac{\sigma_g^2+\sigma_q^2}{2}} \; u \quad \text{and} \quad y = \sqrt{\frac{\sigma_g^2+\sigma_i^2}{2}} \; w.$$

(41)

This results in

$$\frac{dR_o(\tau)}{dR_i(\tau)} = \frac{(2\ell)^2}{4\pi^2} \; \frac{2}{\sigma_g^2+\sigma_i^2} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} e^{-(x^2+y^2) - [2R_i(\tau)/(\sigma_g^2+\sigma_i^2)]xy} \, dxdy.$$

(42)

To further simplify the intregand, introduce the variables v and γ such that

$$x = h(v,\gamma) = v - \frac{a}{\sqrt{1-a^2}} \gamma \qquad v = x + ay$$

(43)

$$\Rightarrow$$

$$y = k(v,\gamma) = \frac{1}{\sqrt{1-a^2}} \gamma \qquad \gamma = \sqrt{1-a^2} \; y$$

(44)

where

$$a = \frac{R_i(\tau)}{\sigma_g^2+\sigma_i^2} \; .$$

(45)

The Jacobian of this transformation is given by

$$J(v,\gamma) = \begin{vmatrix} \dfrac{\partial h}{\partial v} & \dfrac{\partial h}{\partial \gamma} \\ \dfrac{\partial k}{\partial v} & \dfrac{\partial k}{\partial \gamma} \end{vmatrix} = \begin{vmatrix} 1 & \dfrac{-a}{\sqrt{1-a^2}} \\ 0 & \dfrac{1}{\sqrt{1-a^2}} \end{vmatrix} = \frac{1}{\sqrt{1-a^2}} \; .$$

(46)

Hence,

$$dxdy = \frac{1}{\sqrt{1-a^2}} \; dv \; d\gamma$$

(47)

With this transformation, the exponent of the integrand becomes

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$$- [x^2+y^2 + 2a\ xy] = - [v^2 - \frac{2a}{\sqrt{1-a^2}}\ v\gamma + \frac{a^2}{1-a^2}\ \gamma^2 + \frac{2a}{\sqrt{1-a^2}}\ v\gamma \tag{48}$$

$$+ \frac{1}{1-a^2}\ \gamma^2 - \frac{2a^2}{1-a^2}\ \gamma^2] = - \left[v^2 + \frac{a^2+1-2a^2}{1-a^2}\ \gamma^2\right] \tag{49}$$

$$= - [v^2+\gamma^2]\ . \tag{50}$$

Thus, the expression for $\dfrac{dR_o(\tau)}{dR_i(\tau)}$ becomes

$$\frac{dR_o(\tau)}{dR_i(\tau)} = \frac{(2\ell)^2}{4\pi^2}\ \frac{2}{\sigma_g^2+\sigma_i^2}\ \frac{1}{\sqrt{1-a^2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(v^2+\gamma^2)}\ dv d\gamma . \tag{51}$$

The double integrand is readily evaluated to yield

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(v^2+\gamma^2)}\ dv d\gamma = \pi . \tag{52}$$

Consequently,

$$\frac{dR_o(\tau)}{dR_i(\tau)} = \frac{(2\ell)^2}{2\pi}\ \frac{1}{\sigma_g^2+\sigma_i^2}\ \left\{1- \left[\frac{R_i(\tau)}{\sigma_g^2+\sigma_i^2}\right]^2\right\}^{-1/2} \tag{53}$$

Recalling that

$$\frac{d}{dx}\ [\sin^{-1} u] = \frac{1}{\sqrt{1-u^2}}\ \frac{du}{dx}\ , \tag{54}$$

it follows that

$$R_o(\tau) = \frac{(2\ell)^2}{2\pi}\ \sin^{-1} \left[\frac{R_i(\tau)}{\sigma_g^2+\sigma_i^2}\right] + c \tag{55}$$

where c is a constant of integration. To evaluate c, consider the

special case for which

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$$\sigma_g^2 = \sigma_i^2 \qquad (56)$$

and $\tau = 0$. Then

$$R_i(0) = \sigma_i^2 \qquad (57)$$

and

$$R_o(0) = \frac{(2\ell)^2}{2\pi} \sin^{-1}\left(\frac{1}{2}\right) + c \qquad (58)$$

$$= \frac{4\ell^2}{2\pi} \frac{\pi}{6} + c = \frac{\ell^2}{3} + c. \qquad (59)$$

However, $\alpha = \dfrac{\sigma_g}{\sigma_i} = 1$. From page 4 of the notes, the output pdf is

$$P_{V_o}(v_o) = \begin{cases} \dfrac{1}{2\ell} \; ; \; -\ell \le v_o \le \ell \\[2ex] 0 \; ; \; \text{otherwise.} \end{cases} \qquad (60)$$

Hence,

$$E[V_o^2] = R_o(0) = \int_{-\ell}^{\ell} \frac{v_o^2}{2\ell} dv_o = \frac{1}{2\ell} \frac{v_o^2}{3}\Bigg]_{-\ell}^{\ell} = \frac{2\ell^3}{6\ell} = \frac{\ell^2}{3}. \qquad (61)$$

It follows that $c = o$. The output correlation function, therefore, is given by

$$R_o(\tau) = \frac{(2\ell)^2}{2\pi} \sin^{-1}\left[\frac{R_i(\tau)}{\sigma_g^2 + \sigma_i^2}\right]. \qquad (62)$$

Recalling that

$$\rho_i(\tau) = \frac{R_i(\tau)}{\sigma_i^2} \quad \text{and} \quad \alpha = \frac{\sigma_g}{\sigma_i}, \qquad (63)$$

$R_o(\tau)$ can also be expressed as

$$\qquad (64)$$

$$R_o(\tau) = \frac{(2\ell)^2}{2\pi} \sin^{-1}\left[\frac{\rho_i(\tau)}{1+\alpha^2}\right].$$

4. DISTORTION OF THE POWER SPECTRAL DENSITY CAUSED BY THE NONLINEARITY

The power spectral density at the output of the nonlinearity is

given by

$$S_O(\omega) = \int_{-\infty}^{\infty} R_O(\tau) \, e^{-j\omega\tau} \, d\tau \qquad (65)$$

$$= \frac{(2\ell^2)}{2\pi} \int_{-\infty}^{\infty} \sin^{-1}\left[\frac{\rho_i(\tau)}{1+\alpha^2}\right] e^{-j\omega\tau} \, d\tau. \qquad (66)$$

Assume the normalized input correlation function is

$$\rho_i(\tau) = e^{-B|\tau|}. \qquad (67)$$

This corresponds to white Gaussian noise passed through a lowpass RC filter of bandwidth B. Hence, the power spectral density at the nonlinear input is

$$S_i(\omega) = \frac{2B}{B^2+\omega^2} = \frac{2}{B} \frac{1}{1+(\omega/B)^2} \qquad (68)$$

To evaluate the output power spectral density, we expand $\sin^{-1}[\rho_i(\tau)/(1+\alpha^2)]$ into a Taylor series and integrate term by term. For simplicity, let $\alpha = 1$.

Then

$$\sin^{-1}\left[\frac{\rho_i(\tau)}{2}\right] = \frac{\rho_i(\tau)}{2} + \frac{1}{6}\left[\frac{\rho_i(\tau)}{2}\right]^3 + \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{1}{5}\right)\left[\frac{\rho_i(\tau)}{2}\right]^5 + \ldots \qquad (69)$$

$$= \frac{1}{2} e^{-B|\tau|} + .0208 \, e^{-3B|\tau|} + .00234 \, e^{-5B|\tau|} + \ldots \quad . \qquad (70)$$

By inspection, the output power spectral density is given by the series expansion

$$S_O(\omega) = \frac{(2\ell)^2}{2\pi}\left[\frac{1}{2} \frac{2B}{B^2+\omega^2} + .0208 \frac{6B}{(3B)^2+\omega^2} + .00234 \frac{10B}{(5B)^2+(\omega)^2} + \ldots\right] (71)$$

$$= \frac{(2\ell)^2}{2\pi}\left[\frac{1}{B} \frac{1}{1+(\omega/B)^2} + \frac{.014}{B} \frac{1}{1+(\omega/3B)^2} + \frac{.000936}{B} \frac{1}{1+(\omega/5B)^2} + \ldots\right] (72)$$

$$= \frac{(2\ell)^2}{2\pi} \frac{1}{B} \left[\frac{1}{1+(\omega/B)^2} + .014 \frac{1}{1+(\omega/3B)^2} + .000936 \frac{1}{1+(\omega/5B)^2} + \ldots \right] \quad (73)$$

In general, the output spectrum can be thought of as resulting from the superposition of an infinite number of spectra having bandwidths B, 3B, 5B, etc., as shown below.
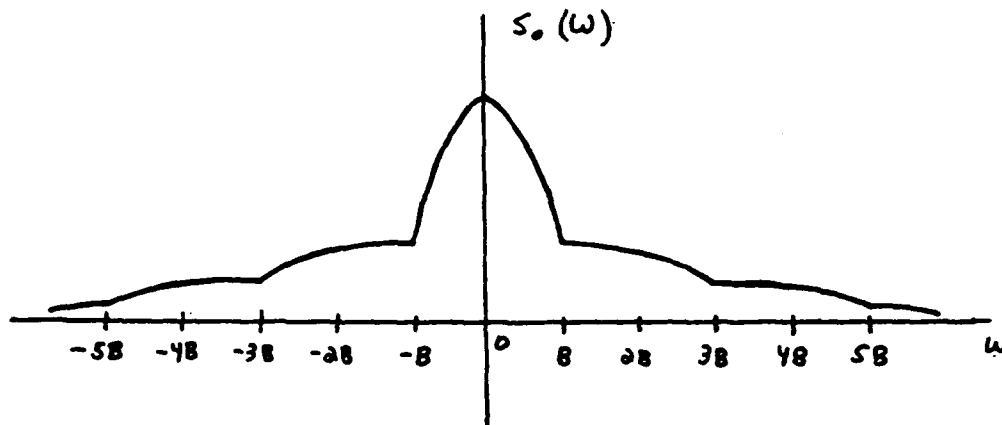
Because the infinite series converages rapidly, the total power outside the interval (-B, B) is relatively small. Nevertheless, it may be significant in the subclutter visibility application.

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Effects of Mutual Coupling on the Angle of Arrival Estimation Problem

Braham Himed   Donald D. Weiner

ECE Dept., Syracuse University
Syracuse, N.Y 13244.
(315) 443-4406

Abstract

This paper deals with the compensation of mutual coupling effects in a sensor array when using the subspace approach known as the matrix pencil. This method is easy to implement and is applicable even in the presence of coherent sources. Computer simulation shows that use of the mutual impedance matrix effectively compensates for the presence of mutuals.

Introduction

Direction finding, which involves the estimation of the angles of arrival of sources, is very important in many sensor systems such as radar, sonar, seismology, etc. Over the years several methods have been proposed to solve this kind of problem [1,2]. Recently, subspace approaches have been introduced [2-7]. They are based on an eigenvalue-eigenvector decomposition of the correlation matrix. They are shown to yield asymptotically unbiased estimates of such signal parameters as angles of arrival and number of signals. However, these methods have not taken into account effects of mutual coupling between array elements which can significantly alter the eigensystems underlying the solution procedures. In this paper, we are dealing with the compensation for mutual coupling effects when using the subspace approach known as the matrix pencil approach. Computer simulations show that significant improvement in the estimates for the angles of arrival can be achieved.

Moving Window.

Assume we have a linear array composed of m identical sensors with uniform spacing D. Let there be $d \leq m$ narrowband sources located at azimuthal angles θ_k ; $k=1,2,\ldots,d$, emitting signals whose complex envelopes are denoted by $s_k(t)$; $k=1,2,3,\ldots,d$. Assume the sources are in the far field such that planar wavefronts arrive at the array. With reference to the first sensor, the received signal at the ith sensor is modeled as

$$y_i(t,\underline{\theta}) = \sum_{k=1}^{d} s_k(t)a(\theta_k)e^{j(i-1)\phi_k} + n_i(t) \qquad (1)$$

for $i=1,\ldots,m,$

where

$n_i(t)$ is the additive noise assumed to be zero-mean,

$\phi_k = \omega D \sin(\theta_k)/c$

ω is th center frequency of each of the spatial sources,

c is the speed of propagation of the plane waves,

$a(\theta)$ is the beam pattern of each sensor.

Given the number of sources d and the m data points , $y_i(t,\underline{\theta})$, we form $(d+1)$ vectors $\underline{Y}_n$ of length $(m-d)$, where

$$\underline{Y}_n^T = (y_n, y_{n+1}, \ldots, y_{n+m-d-1}) \; ; \; n=1,\ldots,(d+1).$$

The two matrices M_1 and N_1 are then formed

$$M_1 = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ | & | & & | \\ \underline{Y}_1 & \underline{Y}_2 & \cdots & \underline{Y}_d \\ | & | & & | \\ \downarrow & \downarrow & & \downarrow \end{bmatrix} \; ; \; N_1 = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ | & | & & | \\ \underline{Y}_2 & \underline{Y}_3 & \cdots & \underline{Y}_{d+1} \\ | & | & & | \\ \downarrow & \downarrow & & \downarrow \end{bmatrix}.$$

Let

$$A = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{j\phi_1} & e^{j\phi_2} & \cdots & e^{j\phi_d} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ e^{j(m-d-1)\phi_1} & e^{j(m-d-1)\phi_2} & \cdots & e^{j(m-d-1)\phi_d} \end{bmatrix}$$

$$B = \begin{bmatrix} a_1 & & & \\ & a_2 & & 0 \\ & & \cdot & \\ & & & \cdot \\ 0 & & & \cdot \\ & & & a_d \end{bmatrix}$$

$$\Phi = \begin{bmatrix} e^{j\phi_1} & & & \\ & e^{j\phi_1} & & 0 \\ & & \cdot & \\ & & & \cdot \\ 0 & & & \cdot \\ & & & e^{j\phi_d} \end{bmatrix}$$

$$\underline{S}^T = (s_1, s_2, \ldots, s_d),$$

and

$$\underline{N}_n^T = (n_n, n_{n+1}, \ldots, n_{n+m-d-1}).$$

Then $\underline{Y}_n$ can be rewritten as

$$\underline{X}_n = A\Phi^{\quad}B\underline{S} + \underline{N}_n \qquad (2)$$

and the matrices M_1 and N_1 become

$$M_1 = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ | & | & & | \\ AB\underline{S} & AB\Phi\underline{S} & \cdots & AB\Phi^{(d-1)}\underline{S} \\ | & | & & | \\ \downarrow & \downarrow & & \downarrow \end{bmatrix} + \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ | & | & & | \\ \underline{N}_1 & \underline{N}_2 & \cdots & \underline{N}_d \\ | & | & & | \\ \downarrow & \downarrow & & \downarrow \end{bmatrix}$$

$$N_1 = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ | & | & & | \\ AB\Phi\underline{S} & AB\Phi^2\underline{S} & \cdots & AB\Phi^d\underline{S} \\ | & | & & | \\ \downarrow & \downarrow & & \downarrow \end{bmatrix} + \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ | & | & & | \\ \underline{N}_2 & \underline{N}_3 & \cdots & \underline{N}_{d+1} \\ | & | & & | \\ \downarrow & \downarrow & & \downarrow \end{bmatrix}.$$

Let F, S, U, N', and N'' be the matrices

$$F = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ | & | & & | \\ \underline{S} & \Phi\underline{S} & \cdots & \Phi^{(d-1)}\underline{S} \\ | & | & & | \\ \downarrow & \downarrow & \cdot & \downarrow \end{bmatrix}$$

$$S = \begin{bmatrix} s_1 & & & \\ & s_2 & & 0 \\ & & \cdot & \\ & & & \cdot \\ 0 & & & \cdot \\ & & & & s_d \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & e^{j\Phi_1} & \cdots & e^{j(d-1)\Phi_1} \\ 1 & e^{j\Phi_2} & \cdots & e^{j(d-1)\Phi_2} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ 1 & e^{j\Phi_d} & \cdots & e^{j(d-1)\Phi_d} \end{bmatrix}$$

$$N' = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ | & | & & | \\ \underline{N}_1 & \underline{N}_2 & \cdots & \underline{N}_d \\ | & | & & | \\ \downarrow & \downarrow & & \downarrow \end{bmatrix}$$

$$N'' = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ | & | & & | \\ \underline{N}_2 & \underline{N}_3 & \cdots & \underline{N}_{d+1} \\ | & | & & | \\ \downarrow & \downarrow & & \downarrow \end{bmatrix}.$$

Then
$$M_1 = ABSU + N' \quad \text{and} \quad N_1 = ABS\Phi U + N''. \qquad (3)$$

Assuming the signals and noise to be statistically independent and that the noise components are uncorrelated from sensor to sensor, we get

$$E[M_1^H M_1] = U^H V U + (m-d)\sigma^2 I_d \qquad (4)$$

$$E[N_1^H M_1] = U^H \Phi^H V U + (m-d)\sigma^2 I_{1d} \qquad (5)$$

where I_d is the (dxd) identity matrix , I_{1d} is the matrix

$$I_{1d} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 0 & 1 & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & & & & & \cdot \\ \cdot & \cdot & \cdot & \cdot & & & & & \cdot \\ \cdot & \cdot & \cdot & \cdot & & & & & \cdot \\ 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & 1 \\ 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \end{bmatrix}.$$

and V the matrix

$$V = E[\underline{S}^H B^H A^H AB\underline{S}].$$

Defining

$$F_{pq} = \sum_{i=1}^{m-d} e^{j(i-1)(\Phi_p - \Phi_q)},$$
$$S_{pq} = E[s_q^* s_p],$$
$$a_{pq} = a_q a_p,$$

the matrix V can be written as

$$V = \begin{bmatrix} S_{11}a_{11}F_{11} & S_{21}a_{21}F_{21} & \cdot & \cdot & \cdot & \cdot & S_{d1}a_{d1}F_{d1} \\ S_{12}a_{12}F_{12} & S_{22}a_{22}F_{22} & \cdot & \cdot & \cdot & \cdot & S_{d2}a_{d2}F_{d2} \\ \cdot & \cdot & & & & & \cdot \\ \cdot & \cdot & & & \cdot & & \cdot \\ \cdot & \cdot & & & & & \cdot \\ \cdot & \cdot & & & & & \cdot \\ S_{1d}a_{1d}F_{1d} & S_{2d}a_{2d}F_{2d} & \cdot & \cdot & \cdot & \cdot & S_{dd}a_{dd}F_{dd} \end{bmatrix}$$

Thus,
$$M = E[M_1^H M_1] - (m-d)\sigma^2 I_d = U^H V U \qquad (6)$$

$$N = E[N_1^H M_1] - (m-d)\sigma^2 I_{1d} = U^H \Phi^H V U. \qquad (7)$$

The matrix pencil then becomes

$$M - \lambda N = U^H V U - \lambda U^H \Phi^H V U = U^H(I - \lambda \Phi^H) V U \qquad (8)$$

which satisfies the requirements of the pencil theorem. Hence, the values of λ for which the rank of $M - \lambda N$ decreases by 1 are given by

$$\lambda_k = e^{j\Phi_k} ; \quad k = 1, 2, \ldots, d. \qquad (9)$$

The angles of arrival are given by

$$\theta_k = \sin^{-1}(-jc\ln(\lambda_k)/\omega D); \quad k = 1, 2, \ldots, d. \qquad (10)$$

Mutual Coupling

Consider a linear array of m dipoles uniformly spaced at a distance D. Each dipole is of length t and has a radius r satisfying the condition r<<t. A load is attached to the center gap of each dipole. Assume there are d narrowband signals impinging on the array as planar wavefronts. The voltages induced by the assumed signals on the loads are the outputs of the dipoles. Induced currents will appear on the dipoles. These currents reradiate and generate scattered fields. The scat-

tered fields then induce currents on the neighboring dipoles. The process of induction and reradiation causes the mutual coupling among the dipoles.

Using one sinusoidal expansion and weighting function per dipole, the method of moments [9,10] was used to obtain the matrix of mutuals. Denote the current distribution by $J(z)$ (assuming longitudinal distribution and neglecting all other distributions) and the j-th expansion function by $f_j(z)$.
Then

$$J(z) = \sum_{j=1}^{m} I(j) f_j(z) \qquad (11)$$

where $I(j)$ are unknown amplitudes to be determined. At a point (y,z) in the Y-Z plane, the scattered field is given by

$$E^{(s)}(y,z) = \sum_{j=1}^{m} I(j) E_j^{(s)}(y,z) \qquad (12)$$

where $E_j^{(s)}(y,z)$ is the scattered field from the j-th dipole. The total field will then be

$$E(y,z) = E^{(inc)}(y,z) + E^{(s)}(y,z) \qquad (13)$$

where $E^{(inc)}$ is the incident field. Let E_z be the z-component of the total field. A generalized voltage $V(i)$ induced on the subsection spanned by the function $f_j(z)$ can be defined with respect to a weighting function $w(z)$ as

$$V(i) = F(E_z(y,z), w_i(z)) \qquad (14)$$

where F is bilinear with respect to $E_z(y,z)$ and $w_i(y,z)$.
Similarly, we define

$$v^{(inc)}(i) = F(E_z^{(inc)}(y,z), w_i(z)) \qquad (15)$$
and
$$v^{(s)}(i) = F(E_z^{(s)}(y,z), w_i(z)). \qquad (16)$$
Thus,
$V(i) = v^{(inc)}(i) + v^{(s)}(i)$, which becomes, for metallic scatterers,

$$V(i) = v^{(inc)}(i) + v^{(s)}(i) = 0$$
or
$$v^{(inc)}(i) = -v^{(s)}(i). \qquad (17)$$

However,

$$v^{(s)}(i) = F(\sum_{j=1}^{m} I(j) E_j^{(s)}(y,z), w_i(z))$$

$$= \sum_{j=1}^{m} I(j) F(E_j^{(s)}(y,z), w_i(z)).$$

Let z^{ij} be

$$z^{ij} = -F(E_j^{(s)}(y,z), w_i(z)). \qquad (18)$$

Thus

$$v^{(s)}(i) = \sum_{j=1}^{m} -z^{ij} I(j) ; \quad i=1,2,\ldots,m. \qquad (19)$$

In matrix notation

$$\underline{v}^{(s)} = -Z \underline{I} \qquad (20)$$

where

$$\underline{v}^{(s)T} = [v^{(s)}(1), v^{(s)}(2), \ldots, v^{(s)}(m)]$$
and
$$\underline{I}^T = [I(1), I(2), \ldots, I(m)].$$

The matrix Z can be decomposed into two parts as

$$Z = Z_0 + Z_L$$
where
Z_0 is the generalized impedance matrix
and
Z_L is the load matrix.

Assuming that all loads are loaded with the same load z_1, the matrix Z_L is given by

$$Z_L = \begin{bmatrix} z_1 & & & \\ & z_1 & & 0 \\ & & \cdot & \\ & & & \cdot \\ & & & \cdot \\ 0 & & & \cdot \\ & & & z_1 \end{bmatrix}.$$

The ij-th element of Z, therefore is

$$z^{ij} = z_{ij} + z_1 \delta_{ij}.$$

The voltages induced on a load z_1 are given by

$$\underline{v}^{(t)} = Z_L \underline{I} \text{ and } \underline{I} = Z_L^{-1} \underline{v}^{(t)}.$$

However,

$$\underline{v}^{(inc)} = Z\underline{I} = Z_0 Z_L^{-1} \underline{v}^{(t)} + \underline{v}^{(t)},$$

which implies that

$$\underline{v}^{(t)} = [I + Z_0 Z_L^{-1}]^{-1} \underline{v}^{(inc)}. \qquad (21)$$

Let H be the matrix

$$H = [I + Z_0 Z_L^{-1}]. \qquad (22)$$

H can be written as

$$H = \begin{bmatrix} 1+(z_{11}/z_1) & (z_{12}/z_1) & \cdots & (z_{1m}/z_1) \\ (z_{21}/z_1) & 1+(z_{22}/z_1) & \cdots & (z_{2m}/z_1) \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ (z_{m1}/z_1) & (z_{m2}/z_1) & \cdots & 1+(z_{mm}/z_1) \end{bmatrix}.$$

Thus, when incident signals are impinging on the array and in the presence of additive noise, the output of the linear array will be

$$\underline{v}^{(t)} = H^{-1} \underline{v}^{(inc)} + \underline{N}.$$

For simplicity, let
$$\underline{X} = \underline{v}^{(inc)}$$
and
$$\underline{Y} = \underline{v}^{(t)}.$$
We now have a relationship between the incident signals and the received signals at the outputs of the array, which is

$$\underline{Y} = H^{-1} \underline{X} + \underline{N}. \qquad (23)$$

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If we try to use the vector $\underline{Y}$ in the formulation of the pencil theorem, it is not possible to obtain the decomposition needed in that formulation. An estimate $\underline{\hat{X}}$ of $\underline{X}$ is needed in order to obtain the decompositions needed in formulation of the pencil theorem.

Assuming that the signals and noise are statistically independent and that the noise components are uncorrelated zero-mean random variables with variance σ^2, the minimum error mean-squared linear estimator results when the error $(\underline{\hat{X}}-\underline{X})$ is orthogonal to the observed data $\underline{Y}$. Let $\underline{\hat{X}} = R \underline{Y}$, where R is to be determined. After some computation and defining C to be the correlation matrix of the observed data $\underline{Y}$, we obtain

$$R = H \, (C-\sigma^2 \, I_m) \, C^{-1}. \tag{24}$$

Computer Simulation

The scenario used for this simulation consisted of two coherent sources ($d=2$) which are incident on a linear array consisting of eight half wavelength dipoles ($m=8$). The sources are assumed to be located at $\theta_1=18°$ and $\theta_2=22°$. The noise was simulated to be white Gaussian with zero-mean and unit variance. The sensors were positioned at half wavelength apart such that $\omega D/c = \pi$. The results of the simulation are shown below.

(Without compensation for the mutuals)

SNR	mean θ_1	mean θ_2	variance θ_1	variance θ_2
30 dB	20.6537	41.4553	4.177E-2	1.98405
25 dB	20.6478	41.5294	4.556E-2	2.98911
20 dB	20.6374	41.6539	5.787E-2	7.64255
15 dB	20.6205	41.1151	1.192E-1	44.5549

(100 snapshots/run, 50 runs)

(With compensation for the mutuals)

SNR	mean θ_1	mean θ_2	variance θ_1	variance θ_2
30 dB	17.9308	21.9435	5.164E-3	8.784E-3
25 dB	17.8171	21.7513	1.947 E-2	3.909E-2
20 dB	17.5739	20.9331	1.486 E-1	2.019E-1
15 dB	14.1798	20.1921	15.63312	5.963E-2

(100 snapshots/run, 50 runs)

Note that extremely poor estimates are obtained without compensation for the mutuals. Compensation results in significant improvement.

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Angle of Arrival Estimation for
Wideband Sources

Braham Himed and Donald D. Weiner
ECE Dept., Syracuse University
Syracuse, NY 13244
(315)443-4406

Abstract

The moving window [1] is one of several operators that can be used in conjunction with the matrix pencil approach for estimating the locations of multiple sources. A major advantage of the moving window is that the locations of d (d<m) __completely correlated__ narrowband sources can be estimated using a linear equispaced array of m sensors. In this paper, the concept of the moving window is extended to the wideband case by using the Fourier series approach to convert each wideband signal into a sum of narrowband signals. A transformation matrix is then introduced to generate a single equivalent narrowband signal. Any narrowband high resolution technique can be used at this point to generate the estimates. Computer simulations show that the moving window performs better than ESPRIT.

Introduction

Many high resolution algorithms have been developed for the case of narrowband signals. However, narrowband modeling is not appropriate for wideband signals. In particular, the time delay cannot be approximated by a phase shift. Recently, several well known narrowband high resolution algorithms have been extended to the wideband case [2-6].

An effective approach consists of decomposing the wideband signals into sums of narrowband signals [3-5]. Wax et. al. [3] use the MUSIC algorithm to obtain a spectral estimate by averaging either geometrically or arithmetically the contributions from each of the subbands. This incoherent processing is referred to as post averaging. Wang and Kaveh [4] have proposed a pre-processing scheme using linear transformations to combine the various sub-bands into a single band. As in the previous paper, the MUSIC algorithm is employed. However, now only one eigendecomposition is needed to estimate the angles of arrival. This approach known as coherent signal subspace processing (CSS) is shown to outperform the previous one. However, a disadvantage is that a preliminary crude estimate of the emitter locations is required. If the angles are clustered within a beamwidth, the method performs well. Otherwise, spatial prefiltering is needed. The contribution of this paper is to show that CSS can be combined with the moving window operator [1] to solve for the angles of arrival. Computer simulations reveal that the newly proposed technique performs better than ESPRIT [7].

Problem Formulation

Consider the problem of estimating the angles of arrival of wideband signals. The notion of Fourier coefficients is used here in conjunction with the matrix pencil approach. Assume that all incoming signals have approximately the same bandwidth B. Let T be an observation interval and let f_L and f_H be the lowest and highest frequencies contained in B. In practice, this band is determined by Fourier decomposition of the received signals. Assume we have a linear array composed of m identical wideband sensors uniformly spaced at a distance Δ. Let there be d (d<m) wideband sources located in the far field so that plane waves arrive at the array. The received signal at the i-th sensor can be modeled as

$$x_i(t) = \sum_{k=1}^{d} a(\theta_k) s_k(t-\tau_{ik}) + n_i(t); \quad i=1,\ldots,m, \quad (1)$$

where τ_{ik} is the time delay that source k takes to travel from the reference point to the i-th sensor. Taking the reference as the first sensor, τ_{ik} can be written as

$$\tau_{ik} = (i-1)(\Delta/c)\sin(\theta_k) \quad (2)$$

where c is the speed of propagation of the waves. Define the Fourier coefficients of the signal received at the i-th sensor by

$$X_i(\omega_r) = (T)^{\frac{1}{2}} \int_{-T/2}^{T/2} x_i(t)\exp[-j\omega_r t] \, dt, \quad (3)$$

where

$$\omega_r = (2\pi/T)(r_1+r), \quad r=1,2,\ldots,R,$$

r_1 is a constant and R is the number of sub-bands.

With reference to equation (1), the r-th Fourier coefficient of $x_i(t)$ can be expressed as

$$X_i(\omega_r) = \sum_{k=1}^{d} a(\theta_k) S_k(\omega_r) e^{-j(i-1)\phi_k(\omega_r)} + N_i(\omega_r) \quad (4)$$

$$\text{for } i=1,2,\ldots,m,$$

where the r-th Fourier coefficients of $s_k(t)$ and $n_i(t)$ are denoted by $S_k(\omega_r)$ and $N_i(\omega_r)$, respectively, and

$$\phi_k(\omega_r) = -(\omega_r)(\Delta/c)\sin(\theta_k). \quad (5)$$

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(d+1) vectors $\underline{X}_n(\omega_r)$ of length (m-d) are formed where

$$\underline{X}_n(\omega_r)=[X_n(\omega_r) \cdots X_{n+m-d-1}(\omega_r)]^T ,$$

$$n=1, 2 \ldots,(d+1)$$

It is easy to show that $\underline{X}_n(\omega_r)$ can be put in the form

$$\underline{X}_n(\omega_r)=A(\omega_r)\phi^{(n-1)}(\omega_r)B\underline{S}(\omega_r)+\underline{N}_n(\omega_r), \qquad (6)$$

where $A(\omega_r)$, $\phi(\omega_r)$, B, $\underline{S}(\omega_r)$ and $\underline{N}_n(\omega_r)$ are

$$A=\begin{bmatrix} 1 & \cdots & 1 \\ e^{j\phi_1(\omega_r)} & \cdots & e^{j\phi_d(\omega_r)} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ e^{j(m-d-1)\phi_1(\omega_r)} & \cdots & e^{j(m-d-1)\phi_d(\omega_r)} \end{bmatrix}$$

$$B=\begin{bmatrix} a_1 & & & \\ & a_2 & & 0 \\ & & \cdot & \\ & & & \cdot \\ 0 & & & \cdot \\ & & & & a_d \end{bmatrix}$$

$$\phi=\begin{bmatrix} e^{j\phi_1(\omega_r)} & & & \\ & e^{j\phi_2(\omega_r)} & & 0 \\ & & \cdot & \\ & & & \cdot \\ 0 & & & \cdot \\ & & & & e^{j\phi_d(\omega_r)} \end{bmatrix}$$

$$\underline{S}^T = [S_1(\omega_r) \; S_2(\omega_r) \cdots S_d(\omega_r)],$$

and

$$\underline{N}_n^T = [N_n(\omega_r) \cdots N_{n+m-d-1}(\omega_r)].$$

Transformation matrices $T_n(\omega_r)$ [3] are then used in such a way that

$$T_n(\omega_r)A(\omega_r)\phi^{(n-1)}(\omega_r)=A(\omega_0)\phi^{(n-1)}(\omega_0) \qquad (7)$$

where ω_0 is a conveniently chosen frequency, usually selected as being the midband frequency. Then

$$T_n(\omega_r)\underline{X}_n(\omega_r)=T_n(\omega_r)A(\omega_r)\phi^{(n-1)}(\omega_r)B\underline{S}(\omega_r)$$

$$+ T_n(\omega_r)\underline{N}_n(\omega_r)$$

$$=A(\omega_0)\phi^{(n-1)}(\omega_0)B\underline{S}(\omega_r)+T_n(\omega_r)\underline{N}_n(\omega_r). \qquad (8)$$

Assuming that we have R sub-bands, we form the arithmetic average as given by

$$\underline{X}_n(\omega_0) = (1/R) \sum_{r=1}^{R} T_n(\omega_r)\underline{X}_n(\omega_r). \qquad (9)$$

Let $\underline{S}'$ and $\underline{N}_n'$ be

$$\underline{S}' = (1/R) \sum_{r=1}^{R} T_n(\omega_r)\underline{S}(\omega_r) , \qquad (10)$$

$$\underline{N}_n' = (1/R) \sum_{r=1}^{R} T_n(\omega_r)\underline{N}_n(\omega_r) . \qquad (11)$$

Therefore, $\underline{X}_n(\omega_0)$ can be expressed as

$$\underline{X}_n(\omega_0)=A(\omega_0)\phi^{(n-1)}(\omega_0)B\underline{S}'+\underline{N}_n'. \qquad (12)$$

In the remainder of this paper the dependence on ω_0 is assumed. The two matrices M_1 and N_1 are then formed where

$$M_1 = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ | & | & & | \\ \underline{X}_1 & \underline{X}_2 & \cdots & \underline{X}_d \\ | & | & & | \\ \downarrow & \downarrow & & \downarrow \end{bmatrix} ; \quad N_1 = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ | & | & & | \\ \underline{X}_2 & \underline{X}_3 & \cdots & \underline{X}_{d+1} \\ | & | & & | \\ \downarrow & \downarrow & & \downarrow \end{bmatrix} .$$

These can also be written as

$$M_1 = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ | & | & & | \\ ABS & AB\phi\underline{S} & \cdots & AB\phi^{(d-1)}\underline{S} \\ | & | & & | \\ \downarrow & \downarrow & & \downarrow \end{bmatrix} + N' \qquad (13)$$

$$N_1 = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ | & | & & | \\ AB\phi\underline{S} & AB\phi^2\underline{S} & \cdots & AB\phi^d\underline{S} \\ | & | & & | \\ \downarrow & \downarrow & & \downarrow \end{bmatrix} + N'' . \qquad (14)$$

Let F, S, U, N' and N'' be the matrices

$$F = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ | & | & & | \\ \underline{S} & \phi\underline{S} & \cdots & \phi^{(d-1)}\underline{S} \\ | & | & & | \\ \downarrow & \downarrow & & \downarrow \end{bmatrix}$$

$$S = \begin{bmatrix} s_1 & & & \\ & s_2 & & 0 \\ & & \cdot & \\ & & & \cdot \\ 0 & & & \cdot \\ & & & & s_d \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & e^{j\phi_1} & \cdots & e^{j(d-1)\phi_1} \\ 1 & e^{j\phi_2} & \cdots & e^{j(d-1)\phi_2} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ 1 & e^{j\phi_d} & \cdots & e^{j(d-1)\phi_d} \end{bmatrix}$$

$$N' = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ | & | & & | \\ \underline{N}_1' & \underline{N}_2' & \cdots & \underline{N}_d' \\ | & | & & | \\ \downarrow & \downarrow & & \downarrow \end{bmatrix}$$

$$N'' = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ | & | & & | \\ \underline{N}_2' & \underline{N}_3' & \cdots & \underline{N}_{d+1}' \\ | & | & & | \\ \downarrow & \downarrow & & \downarrow \end{bmatrix}.$$

Then

$$M_1 = ABSU + N' \tag{15}$$

and

$$N_1 = ABS\phi U + N''. \tag{16}$$

Assuming that the signals and noise are statistically independent and that the noise components are uncorrelated from sensor to sensor, we get

$$E[M_1^H M_1] = U^H V U + E[N'^H N'] \tag{17}$$

$$E[N_1^H M_1] = U^H \phi^H V U + E[N''^H N']. \tag{18}$$

where V is the matrix $V = E[S^H B^H A^H ABS]$. Defining

$$F_{pq} = \sum_{i=1}^{m-d} e^{j(i-1)(\phi_p - \phi_q)},$$

$$S_{pq} = E[S_q^* S_p],$$

$$a_{pq} = a_q^* s_p,$$

the matrix V can be written as

$$V = \begin{bmatrix} S_{11}a_{11}F_{11} & \cdots & S_{d1}a_{d1}F_{d1} \\ S_{12}a_{12}F_{12} & \cdots & S_{d2}a_{d2}F_{d2} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ S_{1d}a_{1d}F_{1d} & \cdots & S_{dd}a_{dd}F_{dd} \end{bmatrix}.$$

Define the matrices M and N as

$$M = E[M_1^H M_1] - E[N'^H N'] = U^H V U \tag{19}$$

and

$$N = E[N_1^H M_1] - E[N''^H N'] = U^H \phi^H V U. \tag{20}$$

The matrix pencil then becomes

$$M - \lambda N = U^H V U - \lambda U^H \phi^H V U = U^H (I - \lambda \phi^H) V U \tag{21}$$

which satisfies the requirements of the pencil theorem. Hence, the values of λ for which the rank of $M - \lambda N$ decreases by 1 are given by

$$\lambda_k = e^{j\phi_k} \; ; \; k = 1, 2, \ldots, d. \tag{22}$$

The angles of arrival are given by

$$\theta_k = \sin^{-1}\{jc\ln(\lambda_k)/\omega_0\Delta\}; \; k = 1, 2, \ldots, d \tag{23}$$

Simulation

Several possibilities exist for choosing the transformation matrices T_n [7]. It can be shown that a diagonal transformation leads to the simplest analysis. Assuming the sources to be clustered within the proximity of one location β, the transformation matrices $T_n(\omega_r)$ then become

$$T_n(\omega_r) = e^{-j(n-1)(\omega_0 - \omega_r)(\Delta/c)\sin(\beta)} T_1(\omega_r)$$

where

$$T_1(\omega_r) = \text{diag}\{ 1 \; e^{-j(\omega_0 - \omega_r)(\Delta/c)\sin(\beta)} \\ \cdots \; e^{-j(m-d-1)(\omega_0 - \omega_r)(\Delta/c)\sin(\beta)} \}.$$

With this transformation it follows that

$$T_n(\omega_r) A(\omega_r) \phi^{(n-1)}(\omega_r) = A(\omega_0) \phi^{(n-1)}(\omega_0).$$

Assuming that the noise components are uncorrelated from sensor to sensor and from sub-band to sub-band with zero mean and variance σ^2, it can be shown that

$$E[N'^H N'] = R \sigma^2 Id$$

$$E[N''^H N'] = R \sigma^2 I_1 d$$

where Id is the $d \times d$ identity matrix and $I_1 d$ is the matrix

$$I_1 d = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & & 0 \\ 0 & 0 & 1 & 0 & \cdots & & 0 \\ 0 & 0 & 0 & 1 & \cdots & & 0 \\ \cdot & \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & \cdot & \cdot & & & \cdot \\ 0 & 0 & 0 & 0 & \cdots & & 1 \\ 0 & 0 & 0 & 0 & \cdots & & 0 \end{bmatrix}.$$

In the simulations, we have considered a linear array consisting of 8 sensors uniformly spaced at a distance $\Delta = c/(2 f_0)$. Following the example in [7], the two sources were assumed to be located at 16° and 20° and to have ideal rectangular spectra of bandwidth B=40 Hz centered at $f_0 = 100$ Hz. The broadband signals were first decomposed into 33 narrowband components. 100 snapshots were taken for each of the 50 runs.

An initial estimate of 18° was used. Two cases were considered for ESPRIT. In the first case, overlapping subarrays were used. Thus, the two subarrays were composed of the first and last seven sensors, respectively. In the second case, non-overlapping subarrays were used. Here 4 pairs of sensors were generated by using adjacent sensors. The sample mean of the estimates are shown in figures 1 and 3 for the sources at 16° and 22°. The corresponding sample variances are shown in figures 2 and 4 along with the Cramer-Rao lower bound (CRLB). In the figures, the moving window is denoted by (1), ESPRIT (case 1) by 2, and ESPRIT (case 2) by (3). (4) represents the CRLB. For signal to noise ratios greater than 10 dB, the three methods produce comparable results for the sample mean. For SNR lower than 10 dB, moving window appears to have the largest bias. However, for all value of SNR between 5 and 30 dB, the moving window has the smallest variances. It is concluded that the moving window works well for broad band sources when applied in conjunction with CSS.
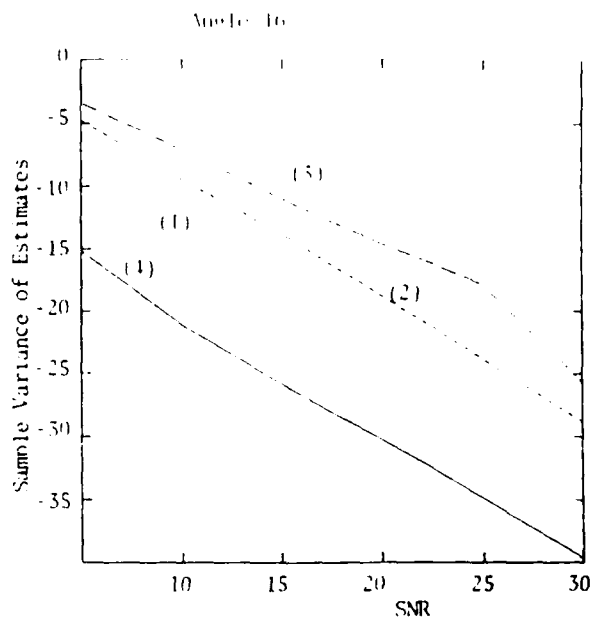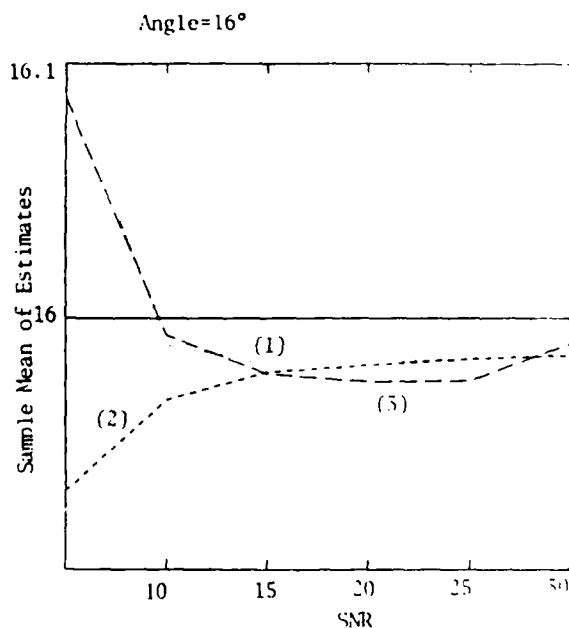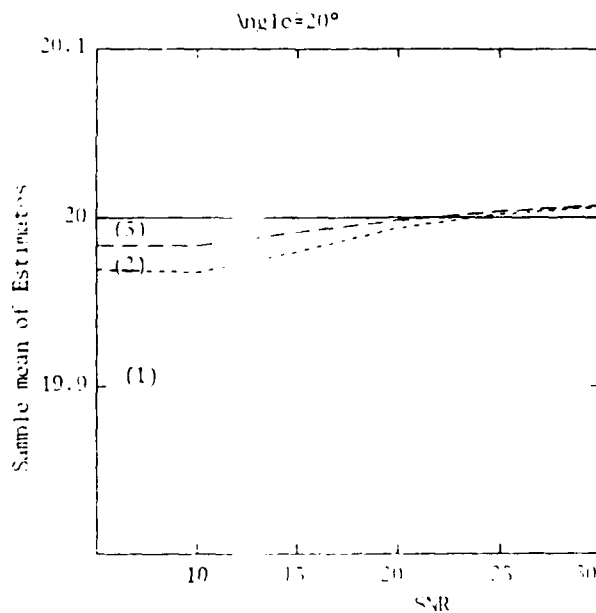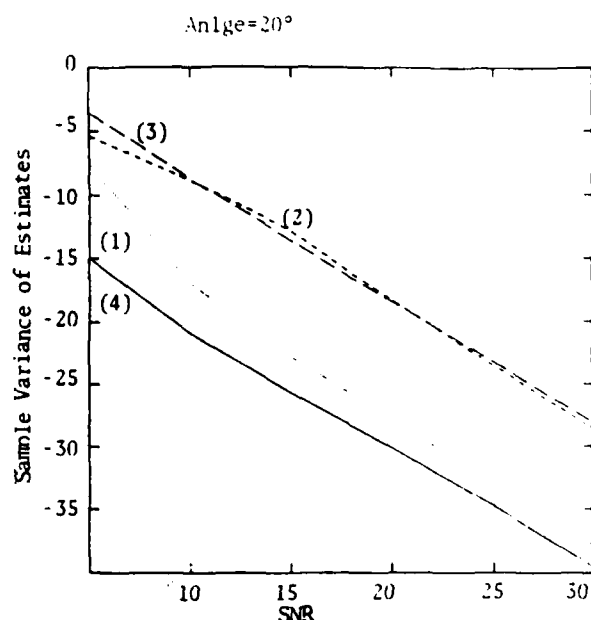

Figure 2

Figure 1

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Anlge=20°

Figure 4

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Use of Phased Arrays for Simultaneous Estimation of Angles of Arrival and Carrier Frequencies

By

Braham Himed   Donald D. Weiner

ECE Dept. Syracuse University.

Several subspace approaches based on phased arrays [1,2,3,4] have been introduced as high resolution direction finding techniques. Typically, the emitted narrowband signals are assumed to have the same carrier frequency. Although the case of unequal carrier frequencies is discussed by [1], a simpler approach, using the moving window operator, is presented in this paper.

Assuming m uniformly spaced sensors, $(m-1)$ delay elements are introduced at the first sensor in conjunction with the pencil approach [2]. Two sets of matrix pencils are generated from the data. Their generalized eigenvalues are shown to be known functions of the carrier frequencies and angles of arrival. Simultaneous solution of these nonlinear equations yields the desired estimates. The method is shown to work well by means of computer simulations.

Assume that the complex envelopes of the emitted signals are stationary random processes. Let the received signal at the ith sensor be

$$y_i(t,\underline{\theta}) = \sum_{k=1}^{d} a(\theta_k) s_k(t)\, e^{j(i-1)[(\omega_k/c)\sin(\theta_k)]} + n_i(t) \quad ;i=1,2,\ldots,m.$$

where D is the sensor spacing, ω_k is the kth carrier frequency, and θ_k is the kth angle of arrival.

The signal received at the $(h+1)^{th}$ delay line tap connected to the first sensor is

$$z_h = y_1(t-hT,\underline{\theta}) = \sum_{k=1}^{d} a(\theta_k) s_k(t) e^{jhT\omega_k} + n_1(t-hT) \quad ;h=0,1,\ldots,m-1.$$

$(d+1)$ vectors $\underline{Y}_n$ and $(d+1)$ vectors $\underline{Z}_n$ are then formed, where

$$\underline{Y}_n^T = (y_n\ y_{n+1}\ \cdots\ y_{n+m-d-1}) \text{ and } \underline{Z}_n^T = (z_{n-1}\ z_n\ \cdots\ z_{n+m-d-2});$$

$$n=1,2,\ldots,d+1.$$

Define the inner products $m_{h,k}$ and $v_{i,n}$ to be

$$m_{h,k}=< \underline{Y}_h,\underline{Y}_k > \text{ and } v_{i,n}=< \underline{Z}_i,\underline{Z}_n >.$$

Four matrices M_1, N_1, P_1 and Q_1 are then formed as follows:

$$M_1 = \begin{bmatrix} m_{1,1} & m_{1,2} & \cdots & m_{1,d} \\ m_{2,1} & m_{2,2} & \cdots & m_{2,d} \\ \vdots & \vdots & & \vdots \\ m_{d,1} & m_{d,2} & \cdots & m_{d,d} \end{bmatrix} \qquad N_1 = \begin{bmatrix} m_{2,1} & m_{2,2} & \cdots & m_{2,d} \\ m_{3,1} & m_{3,2} & \cdots & m_{3,d} \\ \vdots & \vdots & & \vdots \\ m_{d+1,1} & m_{d+1,2} & \cdots & m_{d+1,d} \end{bmatrix}.$$

P_1 and Q_1 are obtained from M_1 and N_1 simply by replacing m with v.

Let I be the Identity matrix and I_1 the matrix defined by

$$I_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

Define the matrices M and N as

$$M = M_1 - (m-d)\sigma^2 I \text{ and } N = N_1 - (m-d)\sigma^2 I_1$$

where σ^2 is the known noise variance.

It can be shown that M and N have the following decompositions

$$M = U V U^H \quad \text{and} \quad N = U V \phi^H U^H$$

where ϕ is a diagonal matrix whose i^{th} diagonal element is $e^{j\phi_i}$ and- where $\phi_i=(\omega_i D/c)\sin(\theta_i)$. Hence, the matrix decomposition of the pencil M-λN is given by

$$M-\lambda N = (UVU^H)-\lambda(UV\phi^H U^H) = UV(I-\lambda\phi^H)U^H.$$

Therefore, the generalized eigenvalues of the pencil M-λN are given by

$$\lambda_i = e^{j\phi_i} \; ; i=1,2,\ldots,d. \qquad (1)$$

Now let P and Q be the matrices

$$P = P_1 + (m-d)\sigma^2 I \quad \text{and} \quad Q = Q_1 - (m-d)\sigma^2 I_1.$$

It can be shown that P and Q can be decomposed as

$$P = X V X^H \quad \text{and} \quad Q = X V \psi^H X^H,$$

The matrix pencil P-δQ then becomes

$$P-\delta Q = (XVX^H)-\delta(XV\psi^H X^H) = XV(I-\delta\psi^H)X^H,$$

where ψ is a diagonal matrix whose i^{th} diagonal element is $e^{jT\omega_i}$.

Hence, the generalized eigenvalues of the pencil P-δQ are given by

$$\delta_i = e^{j\psi_i} = e^{jT\omega_i} \; ; i=1,2,\ldots,d. \qquad (2)$$

Equation (1) together with equation (2) allows us to estimate simultaneously the angular frequencies and the angles of arrival of the sources; i.e

$$\omega_i = jln(\delta_i)/T$$

$$\theta_i = \sin^{-1}[(jln(\lambda_i))/(\omega_i D)]; \; i=1,2,\ldots,d.$$

COMPUTER SIMULATION

The model used in the simulation consisted of two coherent sources (d=2) incident on a linear array of eight omnidirectional uniformly spaced sensors (m=8). The noise was simulated as white. Gaussian with zero-mean and variance $\sigma^2 = 1$. The sources were assumed to be located at $\theta_1=18°$ and $\theta_2=22°$ with center frequencies given by $\omega_1=0.2\times2\pi$ rd/s and $\omega_2=0.25\times2\pi$ rd/s, respectively. D and T were assumed to be equal to 6×10^8 meters and 1 second, respectively.

The results of the simulation are tabulated below.

SNR	ω_1	ω_2	θ_1	θ_2
30 dB	$0.2001\times2\pi$	$0.2495\times6\pi$	18.0012	22.0233
10 dB	$0.1925\times2\pi$	$0.2448\times6\pi$	18.5130	22.7785

Mean of θ_i and ω_i

(500 snapshots/run , 10 runs)

SNR	ω_1	ω_2	θ_1	θ_2
30 dB	$0.3687 \, 10^{-4}$	$0.0546 \, 10^{-4}$	0.0111	0.0045
10 dB	0.0033	0.0004	1.0504	0.4662

Variance of θ_i and ω_i

(500 snapshots/run , 10 runs)

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